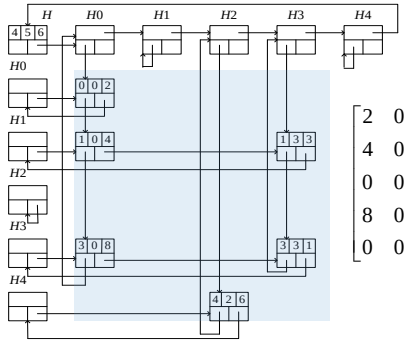
4.9 Sparse Matrices

Sparse Matrix

- A matrix has many **zero** elements.
- Devise a sequential array
  - store **non-zero** elements
  - **row-major** order.
- Access specific column is difficult!
- Using circular lists representation.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

Linked Sparse Matrix



$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

Linked Structure

- Header node: for each row or column
  - **Down**: link to the 1st non-zero term in the column
  - **Right**: link to the 1st non-zero term in the row
  - **Next**: link to the next head node
  - The header node for row *i* is also the header node for column *i*

next	
down	right

- Element node, each **non-zero** term that stores
  - Data of **row, col, and value**
  - A **down** field to link to the next non-zero term in the same **column**
  - A **right** field to link to the next non-zero term in the same **row**

row	col	value
down	right	

- The header of header nodes (a circular list)
  - Store dimension of the matrix

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Create a Sparse Matrix

- Given an $n \cdot m$ sparse matrix with r non-zero terms
  - the total number of required nodes are $\max\{n, m\} + r + 1$
- Input format:
  - The 1st line gives the dimension of matrix and # of non-zero terms.
  - Each subsequent input line is a triple of the form (i, j, a_{ij}).
    - Triples are ordered by rows and within rows by columns.

Input
5,4,6;
0,0,2;
1,0,4;
1,3,3;
3,0,8;
3,3,1;
4,2,6;

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

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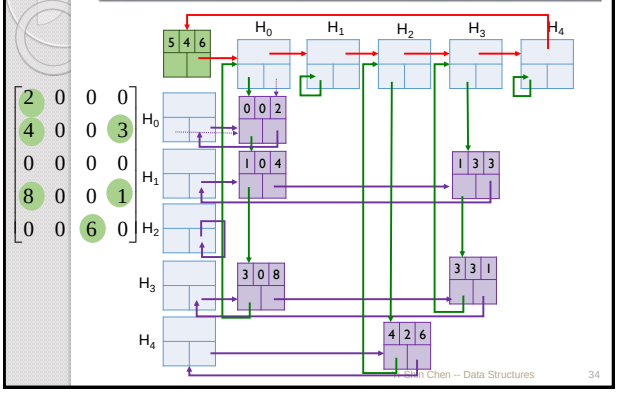
Sparse Matrix in Linked Structure



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Performance analysis: Create a Sparse Matrix

- Performance analysis
  - Set up header nodes, $O(\max\{n, m\})$
  - Set up non-zero nodes, $O(r)$
  - Close row, column lists, $O(\max\{n, m\})$
  - Link header nodes, $O(\max\{n, m\})$
- Total complexity:
  $O(\max\{n, m\} + r) = O(n + m + r)$

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