

**4.9**

**Sparse Matrices**

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### Sparse Matrix

- A matrix has many **zero** elements.
- Devise a sequential array
  - store **non-zero** elements
  - **row-major** order.
- Access specific column is difficult!
- Using circular lists representation.

2	0	0	0
4	0	0	3
0	0	0	0
8	0	0	1
0	0	6	0

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### Linked Sparse Matrix

2	0	0	0
4	0	0	3
0	0	0	0
8	0	0	1
0	0	6	0

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### Linked Structure

- Header node: for each row or column
  - Down: link to the 1<sup>st</sup> non-zero term in the column
  - Right: link to the 1<sup>st</sup> non-zero term in the row
  - Next: link to the next head node
  - The header node for row  $i$  is also the header node for column  $i$
- Element node, each non-zero term that stores
 

row	col	value
down	right	

  - Data of row, col, and value
  - A down field to link to the next non-zero term in the same column
  - A right field to link to the next non-zero term in the same row
- The header of header nodes (a circular list)
  - Store dimension of the matrix

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### Create a Sparse Matrix

- Given an  $n \cdot m$  sparse matrix with  $r$  non-zero terms
  - the total number of required nodes are  $\max\{n, m\} + r + 1$
- Input format:
  - The 1<sup>st</sup> line gives the dimension of matrix and # of non-zero terms.
  - Each subsequent input line is a triple of the form  $(i, j, a_{ij})$ .
    - Triples are ordered by rows and within rows by columns.

<b>Input</b>
5,4,6;
0,0,2;
1,0,4;
1,3,3;
3,0,8;
3,3,1;
4,2,6;

2	0	0	0
4	0	0	3
0	0	0	0
8	0	0	1
0	0	6	0

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### Sparse Matrix in Linked Structure

The diagram illustrates the linked structure of a sparse matrix. On the left, a matrix is shown with non-zero elements highlighted in green: (0,0)=2, (0,3)=3, (1,0)=4, (1,3)=3, (2,0)=8, (2,6)=6, (3,0)=3, (3,3)=1, (4,2)=4, (4,6)=6. Header nodes H<sub>0</sub> through H<sub>4</sub> are shown as boxes with their respective row and column indices. Element nodes are shown as boxes containing their row, column, and value, with arrows indicating their 'down' and 'right' links to the next non-zero term in the same column or row. For example, H<sub>0</sub> points to the first non-zero term in column 0, which is (1,0)=4. This node points right to (1,3)=3, and down to (3,0)=3. The node (3,0)=3 points down to (4,0)=8. The node (1,3)=3 points down to (3,3)=1. The node (4,2)=4 points down to (4,6)=6. The node (4,6)=6 points right to the next header node H<sub>4</sub>.

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**Performance analysis:  
Create a Sparse Matrix**

- Performance analysis
  - Set up header nodes,  $O(\max\{n, m\})$
  - Set up non-zero nodes,  $O(r)$
  - Close row, column lists,  $O(\max\{n, m\})$
  - Link header nodes,  $O(\max\{n, m\})$
- Total complexity:  
 $O(\max\{n, m\} + r) = O(n + m + r)$

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